Commognitive Analysis of Students Difficulty In Solving Fractional Problems

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Abstract

This study discusses the difficulties of elementary school students in completing fractional tasks. Fractions is one of the basic materials in mathematics for elementary school level. The theoretical framework used to analyze students' difficulties in solving fractional tasks is the comognitive framework. The statements in the Comognitive Framework will be broken down into four categories: visual mediators, endorsed narratives, routine and word use. The data was collected through a written test and an interview on the fifth-grade students of a primary school in one of the public schools in Rembang Subdistrict, Pasuruan District. A total of thirty-six students were given tasks related to fractions. The next step is to select one student from each of the high, medium, and low category who answered incorrectly to the question given in the interview. The results of the analysis using comognitive framework shows that the students' difficulties were mostly in the case of errors in visual mediators, endorsed narratives, and routine. These types of errors will affect the completion of the next task. The students are very rarely using visual mediators in completing fractional tasks. Thus, it makes them difficult to complete the tasks on fraction.

Keywords: word use, visual mediator, endorsed narrative, routine.

1. Introduction

Students who have never learned about fractions will generally consider that the characteristics of the operations on fractions are similar to those of integer operations [1]–[3]. However, when student complete tasks on fraction, they discovered that some of properties fractions which they perceive as integers are not. For example, some fractions may refer to the same point on the line of numbers (2/4 is equal to ½ or 4/8)

and similar fractions may refer to completely different objects (1/2 can mean part of the whole or part of a set).Besides, the magnitude or value of the fractions does not match the absolute value of the numerator and denominator [4]. For example, seven is greater than five, and seventeen is greater than ten, but on fractions 7/17 less than 5/10. In addition, the procedure for fractional multiplication is not always consistent as in integer multiplication procedures.

For example, in the multiplication of two fractions will result in a smaller result than multiplication on two integers which will result in bigger results. The error in the fractional calculation problem is possibly due to an error in the application of the principle of integers to fractions or other possibilities because the students have the understanding that the division operation by multiplication may be confusing in addition and subtraction procedures by multiplying the factor, which is true for crossover operations between the numerator and denominator [5]. On the other hand, integers and fractions have the similar basic conceptual structure [6], [7]. For understanding how many times the divisor goes in the dividend can help students see why six divided by 1/3 is 18.

Students also need to develop an understanding of the density of rational numbers, that there is an infinite number of fractions between two consecutive integers eg. between zero and one [8]. When students are asked a question of what lies between zero and one, they might come up with the answers of ½, 1/3 and, ¼. It showed that the answer the students provide are less varied. It is evidence that the student assumption on the number which lies between zero and one are only few numbers. They have not recognized that actually there are indefinite numbers there between zero and one.

The development of numerical knowledge is often seen as a segmented process, in the knowledge of integers is obtained naturally while fractional knowledge needs to encounters a lot of difficulties [3], [9]–[11]. Conceptual change, however, is challenging when new information about fractions appears to be incompatible with the work of the students [12]–[14]. Unfortunately, many students even in high schools and college do not fully understand the difference between integers and fractions [4], so there is a need to think about solutions to solve them.

Thinking is individual communication with oneself [15]. The process of communication interpersonally and cognitive process are of two different manifestations within in similar phenomenon so that [16] combined those two terms, communication and cognition into a new term called as comognition. Frame work comognition as comognitive is used to analyze the learning process of mathematics, [15] introduces four interrelated features of mathematical discourse: the use of words, visual mediators, narrative support, and routines. Word use refers to mathematical vocabulary and syntax and ordinary words which have specific meaning in mathematics. Visual mediators refer to diagrams and symbolic of mathematical objects, as well as physical objects used as teaching aids. Endorsed narrative: refers to a collection of propositions (such as theorems and definitions), evidence, and calculation rules, accepted in a particular mathematical community. Routines mean regularly patterned recurring activities (eg. addition, proof, generalization), as well as a set of meta rules (eg. how to calculate, how to prove, how to generalize). Word use refers to the way the student uses the vocabulary/term in the discussion and discourse, for example ¼ is said to be a quarter, or other examples such as triangle, and function. Visual mediators (visual mediators) refer to mathematical objects including symbols, diagrams, and graphs such as ¼, x^2, dy/dx, <,> which are used for mathematical communication. Habits / routines are rules which govern the actions of the students in discourse or discussion, for example in solving a mathematical problem in the form of a case, students are accustomed to using algebraic equations, students can simplify the equations when it is possible to do so, students know when to use a theorem and when to prove the theorem, or students sketch charts of functions that are accurately assumed although they are not able to describe the scale yet. As for those supporting narratives (endorsed narrative) means referring to what is spoken or written mathematical statements such as $2 + 2 = 4$, (x2) '= 2x, or the number of interior angles on the triangle is 1800 where the student can argue incorrectly or correctly or attributing his opinion to true or false relating to word use, mediators, and routines.

2. Method

The type of research conducted was designed in a qualitative descriptive manner with a qualitative approach.

2.1. Materials

There are two-word question given to students. Both problems were similar in order to see the students'

ability in solving the word problem. The following are the word problem given to student.

1) Dina's money is 2/7 of Bunga's money. If the total of their money is IDR 81,000.00. What is the different between Dina's and Bunga's money?

2) Dina's money is 2/7 of Bunga's money. If the difference in their money is IDR 46,000.00. How much is the total money does Dina and Bunga have?

2.2. Participants and Procedure of Research

Participants in this study were the fifth graders of Mojoparon Elementary School, Rembang Pasuruan, East Java. A fifth grader consists of thirty-six students consisting of 14 male students and 22 female students. Three of thirty-six students were selected as the representative of those for three different categories with high, medium and low achievement. The categories were made based on the number of incorrect answer that the students have given during the interview.

Based on students' work, it can be analyzed using the commognitive framework. To clarify understanding (cognitive) and smoothness of the explanation (communication) interview techniques were used. The interviews were recorded and transcribed. The interview transcripts were selected as important components related to commognitive analysis in solving the words problems.

3. Results and Discussion

Materials of fraction given in the fifth grade of elementary school level is related to the operation on the fraction. The assignment given to students is in the form of cases. The task is given to all students to be completed in a classical manner. Furthermore, the results of students' works were analyzed and categorized into high, medium, low. The result of the analyses was presented in Table 1. The results of students' written test which were grouped in four criteria comognition was presented in Table 2.

Table 1. Result of Student Work

Category	Number of students	Correct answer	Incorrect answer
High	6	16.6%	83.4 %
Moderate	25	12%	98.75%
Low	5	0%	100%

Table 2. Student Answers Within the Comognitive Framework

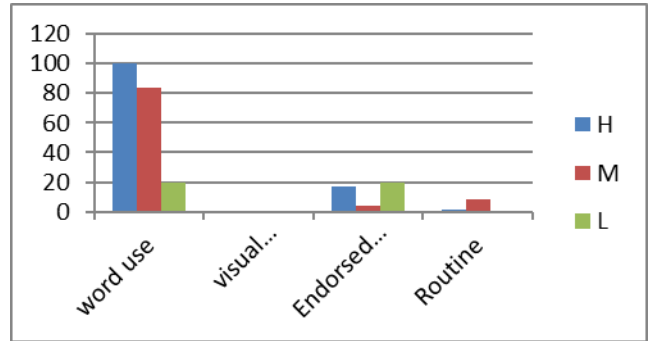| Criteria	Word Use	Visual Mediators	Endorsed Narratives	Routines
High (H)	100 %	0 %	16.6 %	1.6 %
Moderate (M)	84 %	0 %	4 %	8 %
Low (L)	20 %	0 %	20 %	0 %

Fig. 1 Diagram of Student answer within comognitive framework

Based on the results of the students' work, for the students on the high category there is only one student who was able to answer correctly. In the medium category, there were three students who could answer the question correctly. While in the low category, no student was able to answer correctly. Overall only 11.11% of students were able to answer the cases related to the operation of fractions. Based on the results of questionnaires answered by the students, 91. 66% of students responded that they had difficulty when solving the problems of fractional operations. To analyze students' difficulties in completing the tasks on fractions, one student with incorrect answers was selected to represent each category in commognition framework.

Sfard's commognitive framework is a useful framework for analyzing mathematical discourse of students' difficulties in completing fractional tasks. Here we explained how the analysis using of Sfard 's commognitive framework in each category.

3.1. Student high category (S1)

The result of the anaysis on students' difficulties in completing fractional tasks obtained through indepth interviews using commognitive framework can be seen from the interview transcript presented below.

Q: Please explain the meaning of the question by using your own words!

S1: From the question, it was noted that Dina's amount of money was two sevenths of Bunga's amount money.

...

Q: What material is the question about?

S1: Fractions

...

Word Use

This research is related with how students use and process mathematical words such as fractions and two-sevenths. More specifically, it questioned about how students use these words is depending on the context (eg, arguments, illustrations) and how students use synonyms and antonyms to express meaning. During the interview, students were asked to engage in tasks such as making sentences, identifying synonyms, verbalizing definitions, and using words in explaining solutions. Based on the results of the interviews and the student's written work in the category of high-ability, they were able to use the word correctly.

...

Q: How did you solve it?

S1: Using regular calculations like this (while pointing to student answers)

...

Visual Mediators

Along with the words use, the visual mediator serves as a medium for meaning-making. The research is related with how the students use and process mediators associated with the concept of fractions. We categorized modes in the case of symbolic mediators such as cahrt, and algebraic expressions for fractional issues as syntactic, concrete, or objectified. For each mode, participants have the math task to engage in discourse. All the students in the high category have not created any visual mediators when solving the problem, either a chart or sketch.

...

Q: Please explain what is the meaning of your solution, why did you cross it?

S1: Em, ... the answer is incorrect.

Endorsed Narratives

It is important to realize that endorsed narratives can appear in students' responses while not being explicitly articulated. Therefore, we recommended a research design of mathematical assignments which provide an opportunity to make mathematical statements. This

research aimed to find out how students make mathematical arguments related to fractions and ask questions about fractional operations. However, students in high category were not able to give a reason why the answer was incorrect or showed what the correct one was.

Routine

Each efine abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

3.2. Students in the meoderat category (S2)

The results on students in the meoderat category (S2) can be seen in the following interview transcription.

…

Q: Please explain the meaning of the case by using your own words!

S2: From the case it was of noted that Dina's amount of money is two divided by seven of Bunga's amount of money, mm .... two-sevenths of their amount of money.

Q: How did you read it?

S2: It's two-seventh

Q: Why are two-sevenths?

S2: Due to fractions.

...

Word Use

Students in the medium category were able to use the word correctly even though at first students had wrong in the word use.

Visual Mediator

Along with words, the visual mediator serves as a medium for meaning-making. We categorize in the case of symbolic mediators such as cahrt, and algebraic expressions for fractional issues as syntactic, concrete, or objectified. The answer of the students in the medium category yet there is no visual mediator when solving the problem, either on chart or skesta.

Endorsed Narratives

It is important to realize that supported narratives can appear in student's responses while not being explicitly articulated. The answer of the student's work on the category is giving the incorrect narrative. It was not

appropriate or suitable with the given task. It should be the difference that thes students need to provide not the amount of the additions of the money.

Routines

When students use incorrect narrative in solving mathematics problems, then the routines done by students were also incorrect. Therefore, in solving of the problem, it should be by reducing between the denominator not to sum up the numerator and denominator.

3.3. Student Low Category (S3)

The result of the analysis of the students in low category (S3) can be seen from the following interview transcription.
...
Q: Please explain the meaning of the case by using your own words!
S3: From the case it was noted that Dina's amount of money was two-sevenths of Bunga's amount of money.
Q: Why are two-fifths?
S3: Due to fractions.
...

Word Use

Based on the results of the interview, students in the low category have used the appropriate word. Word use used is related to fractions and two-sevenths.

Visual Mediator

Along with words, the visual mediator serves as a medium for meaning-making. Based on the results of the students 'work presented in Figure 4 and interview results, the subjects tend to engage visual mediators in the form of algebra, even though the students' work has not been accurate.

Endorsed Narratives

It is important to realize that supported narratives may appear in student responses while not being explicitly articulated. S3 is using Endorsed narrative, but used by S3 is not right yet. Should be in accordance with the task given, the answer asked is the amount of money, but S3 answered the difference from money.

Routine

Based on the written answer of the assignment given to the student and the result of the S3 interview shows that the routines the students performed by summing between the denominator and the numerator of the given problem. But the routines done by the students have not been exact, not in accordance with what is known and what is being asked.

4. Conclusion

Student difficulties in solving fractional tasks were analyzed using a comognitive framework consisting of word use, visual mediator, endorsed narratives, and routine. Subjects in the high category are correct in the use of words, but errors still occur in the use of visual mediators, endorsed narratives, and routines. In the medium category, the students almost made mistake in word use, but finally they were aware of the problem. In using the visual mediator, endorsed narratives and routines, they were still not able to use the correct ones, while in the low category, the students applied word use.

Visual mediators is an important role as a tool for communication. With routine analysis, researchers can have access to student's thinking that is not so strongly tied to the language. Based on the results of data analysis of student difficulties in the completion of fractional tasks originated from visual mediators, when the selection of the visual mediator is incorrect or even wrong, the student will have difficulty in proceeding on the next solution. Visual mediators can be said as the students' interpretation in completing a task or a mathematical problem. Visual mediators are associated with endorsed narratives, because if student interpretation is incorrect then the argument given by the student is also incorrect. The impact will also affect the student's routine.

Discourse analysis in mathematics should be considered interactions between students and between students and teachers. There are also other elements to be analyzed including interpersonal dynamics in the learning environment and norms of society, including ways the students are encouraged to engage in conversation and express their thoughts. We argue that the essence of discourse analysis, based on a cognitive approach lies in the discourse analysis of students who consider various communicative, verbal and nonverbal languages, exploring past and present meaning through

participation, and clarifying between text, context and culture discourse.

References

[1] Y. Ni and Y.-D. Zhou, "Teaching and learning fraction and rational numbers: The origins and implications of whole number bias," *Educ. Psychol.*, vol. 40, no. 1, pp. 27–52, 2005.

[2] R. S. Siegler, C. A. Thompson, and M. Schneider, "An integrated theory of whole number and fractions development," *Cogn. Psychol.*, vol. 62, no. 4, pp. 273–296, 2011.

[3] M. Irfan, T. Nusantara, S. Subanji, and Sisworo, "Why Did the Students Make Mistakes in Solving Direct and Inverse Proportion Problem?," *Int. J. Insights Math. Teach.*, vol. 01, no. 1, pp. 25–34, 2018.

[4] M. Schneider and R. S. Siegler, "Representations of the Magnitudes of Fractions," *J. Exp. Psychol. Hum. Percept. Perform.*, vol. 36, no. 5, pp. 1227–1238, 2010.

[5] K. J. Newton, C. Willard, and C. Teufel, "AN EXAMINATION OF THE WAYS THAT STUDENTS WITH LEARNING DISABILITIES SOLVE FRACTION COMPUTATION PROBLEMS abstract," *Elem. Sch. J.*, vol. 115, no. 1, pp. 1–21, 2015.

[6] M. W. Alibali and P. G. Sidney, "Variability in the natural number bias : Who , when , how , and why," *Learn. Instr.*, vol. 37, pp. 56–61, 2015.

[7] M. Irfan, T. Nusantara, Subanji, and Sisworo, "Students know the concept but are wrong in solving the proportional problem: How did it happen?," *Int. J. Sci. Math. Technol. Learn.*, vol. 27, no. 1, 2019.

[8] C. L. Smith, G. E. A. Solomon, and S. Carey, "Never getting to zero: Elementary school students' understanding of the infinite divisibility of number and matter," *Cogn. Psychol.*, vol. 51, no. 2, pp. 101–140, 2005.

[9] R. Gelman, "Enabling constraints for cognitive development and learning : Domain Domain . specificity in cognitive development : universals and," no. January 1998, 1998.

[10] D. C. Geary, *Development of Mathematical Understanding*, no. June 2007. 2018.

[11] M. Irfan, T. Nusantara, Subanji, Sisworo, Z. Wijayanto, and S. A. Widodo, "Why do pre-service teachers use the two-variable linear equation system concept to solve the proportion problem?," in *Journal of Physics: Conference Series*, 2019.

[12] J. McMullen, E. Laakkonen, M. Hannula-Sormunen, and E. Lehtinen, "Modeling the developmental trajectories of rational number concept (s)," *Learn. Instr.*, vol. 37, pp. 14–20, 2015.

[13] J. Van Hoof, R. Janssen, L. Verschaffel, and W. Van Dooren, "Inhibiting natural knowledge in fourth graders: towards a comprehensive test instrument," *ZDM*, vol. 47, no. 5, pp. 849–857, 2015.

[14] M. Irfan, T. Nusantara, Subanji, and Sisworo, "Direct Proportion Or Inverse Proportion ? The Occurrence Of Student Thinking Interference," *Int. J. Sci. Technol. Res.*, vol. 8, no. 07, pp. 587–590, 2019.

[15] A. Sfard, "Thinking As Communicating," 2008.

[16] A. Sfard, *Thinking as Communicating*. New York: United States of America by Cambridge University Press, 2008.