SERI MATERI KULIAH

Aljabar Linear Elementer

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ALJABAR LINEAR ELEMENTER

INVERS MATRIKS

MELALUI OBE

Invers Matriks

$A = B^{-1}$

$AB = I$

$ABB^{-1} = IB^{-1}$

$AI = B^{-1}$

$A = B^{-1}$

Misalkan A *adalah matriks bujur sangkar.*

B *dinamakan invers dari* A *jika dipenuhi*

$B = A^{-1}$

$\boxed{AB = I}$ dan $\boxed{BA = I}$ $\rightarrow B^{-1}BA = B^{-1}I$

$A = B^{-1}$

Sebaliknya, A *juga dinamakan invers dari* B.

Notasi $\boxed{A = B^{-1}}$ $\Rightarrow$ $\boxed{B^{-1}B = BB^{-1} = I}$.

Cara menentukan invers suatu matriks A *adalah*

$$(A|I) \overset{\text{OBE}}{\sim} (I|A^{-1})$$

$$(A|I) \overset{\text{OBE}}{\sim} (\ \cdots\ )$$

Jika OBE dari A *tidak dapat menghasilkan matriks identitas maka* A *dikatakan* ***tidak punya invers***

Contoh :

Tentukan matriks invers (jika ada) dari :

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{pmatrix}.$$

$$(A \mid I) \overset{OBE}{\sim} (I \mid A^{-1})$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{lrrrrrr} -3b_1: & -3 & -3 & 0 & 0 & -3 & 0 \\ b_2: & 3 & 2 & -1 & 1 & 0 & 0 \; + \\ \hline & 0 & -1 & -1 & 1 & -3 & 0 \end{array}$$

Jawab :

$$\underbrace{\left(\begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array}\right)}_{A_{3\times3} \qquad I_{3\times3}} \underset{\sim}{b_1 \leftrightarrow b_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 3 & 2 & -1 & 1 & 0 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} -3b_1+b_2 \\ 2b_1+b_3 \end{array}$$

$$\begin{array}{lrrrrrr} 2b_1: & 2 & 2 & 0 & 0 & 2 & 0 \\ b_3: & -2 & -2 & 1 & 0 & 0 & 1 \; + \\ \hline & 0 & 0 & 1 & 0 & 2 & 1 \end{array}$$

$$\begin{array}{l} -3b_1+b_2 \\ 2b_1+b_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ \boxed{0} & \boxed{-1} & -1 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right) -b_2 \qquad \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right)$$

$-1b_3 + b_2$

$-b_3$: 0 0 -1 0 -2 -1

b_2 : 0 1 1 -1 3 0 +

0 1 0 -1 1 -1

$$-b_3 + b_2 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right)$$

$-1b_2 + b_1$

$$-b_2 + b_1 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right)$$

I $\qquad$ A^{-1}

Jadi Invers Matriks A adalah

$$A^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 2 & 1 \end{pmatrix}$$

- **Perhatikan bahwa :**

$$AA^{-1} = \underline{A^{-1}A} = I$$

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{pmatrix} \quad \textbf{dan} \quad A^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 2 & 1 \end{pmatrix}$$

maka

$$AA^{-1} = \begin{array}{cc} A & A^{-1} \\ \begin{pmatrix} \boxed{3 \quad 2 \quad -1} \\ \boxed{1 \quad 1 \quad 0} \\ -2 \quad -2 \quad 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 2 & 1 \end{pmatrix} \end{array} = \begin{pmatrix} 3-2+0 & 0+2-2 & 3-2-1 \\ 1-1+0 & 0+1+0 & 1-1+0 \\ -2+2+0 & 0-2+2 & -2+2+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$

$$A^{-1}A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$

$$\boxed{A^{-1}A = AA^{-1} = I}$$

Berikut ini adalah sifat-sifat matriks invers :

i. $(A^{-1})^{-1} = A$

ii. Jika A, B dapat dibalik atau memiliki invers

maka $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$

iii. Misal $k \in Riil$ maka $(kA)^{-1} = \frac{1}{k} A^{-1}$

iv. Akibat dari (ii) maka $(A^n)^{-1} = (A^{-1})^n$

Terimakasih